BACKREACTION EFFECT IN THE TWO-DIMENSIONAL DILATON GRAVITY

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Abstract

In this work we find static black hole solutions in the context of the two-dimensional dilaton gravity, which is modified by the addition of an R^2 term. This term arises from the one-loop effective action of a massive scalar field in its large mass approximation. The basic feature of this term is that it does not contribute to the Hawking radiation of the classical black hole backgrounds of the model. From this point of view a class of the solutions derived are interpreted as describing backreaction effects. In particular it is argued that evolution of a black hole via non-thermal signals is possible. Nevertheless this evolution seems to be 'soft', in the sence that it does not lead to the evaporation of a black hole, leaving the Hawking radiation as the dominant mechanism for this process.

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1 Introduction

After the initial observation that pure two-dimensional gravity accepts black hole solutions [1], a model was proposed describing the formation of a black hole by the collapse of conformal matter followed by the subsequent emission of Hawking radiation [2]. Furthermore the quantum evolution of the classical black hole background has been studied [3] - [8]. Thus two-dimensional dilaton gravity has been recognized as a usefull theoretical laboratory for the study of the quantum aspects of gravity, and considerable activity persists in the recent years in this field.

In a recent publication [9] the contribution of a massive scalar field to the Hawking radiation rate of a static black hole in the context of the two-dimensional dilaton gravity, was calculated. The calculation shows that the Hawking effect is enhanced in this case, in comparison with that of the conformal matter [2], provided that the mass of the quantized field is less than one, in units of the cosmological constant of the model. Note that this happens to be the case if the scalar field is the tachyon field. The tachyon field is an ordinary massive field in two dimensions as is dictated from the stability requirement of the classical background solutions. It was also recognized in [9] that the thermal radiation effect is due to the coupling of the scalar field to the dilaton. In particular it was shown that terms of the form $R\beta(\Box)R$, in the effective action do not contribute to the Hawking radiation effect. This observation makes interesting the investigation of the evolution of a black hole in the presence of such purely geometric terms in the effective action. The effect of such terms is non thermal and deviation from the thermal evolution yields the possibility of retrieving information from the black hole. Indications about such deviations are already pointed out in the context of the RST model [3, 10].

In this work we address this problem, that is we try to get information about the evolution of a black hole in the presence of such terms. In section 2 we set up the problem justifying that a simple term of the form R^2 can emerge from the one-loop action in a large mass approximation. In section 3 we derive the static black hole solutions of the two-dimensional dilaton gravity after the addition of this term. It is found that the black hole geometries which tend asymptotically to flat space with linear dilaton, can be associated in a certain way to the classical black hole backgrounds. It is argued therefore that these solutions incorporate the backreaction effects due to the

term added in the Lagrangian. Finally in section 4 we discuss our results, supporting further the above argument.

2 Preliminaries

The classical action of a scalar field coupled to the two-dimensional dilaton gravity is

$$S_{cl} = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left\{ \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \left[(\nabla T)^2 - m_0^2 T^2 \right] \right\} \quad (1)$$

where m_0^2 is negative if the scalar field is the tachyon field, and positive in the case of an ordinary massive field. Note that if we rescale the scalar field as $T = e^{-\phi}\widetilde{T}$, then the field $\widetilde{T}$ which has canonical kinetic terms is coupled to the dilaton through the quadratic coupling $-(Q+m^2)\widetilde{T}^2$ where $m^2 = \lambda^2 + m_0^2$, $Q = (\nabla\phi)^2 - \Box\phi - \lambda^2$. In the case of flat space with linear dilaton, $Q = 0$, while stability of the background, under scalar field perturbations, enforces m^2 to be positive. The positivity of m^2 is also required from the stability of the black hole backgrounds. Thus in two dimensions the tachyon field is an ordinary propagating mode.

The non-local part of the one-loop effective action (which is responsible for the Hawking radiation), if we keep only quadratic terms in the background fields Q, and R, is given by [9, 11]

$$\Gamma_{nloc} = \frac{1}{8\pi} \int d^2x \sqrt{g} \left\{ Q\beta^{(1)}(\Box)Q - 2Q\beta^{(2)}(\Box)R + R\beta^{(3)}(\Box)R \right\} \quad (2)$$

where

$$\beta^{(1)}(\Box) = \frac{1}{\Box}\sqrt{\gamma}\, ln\frac{1+\sqrt{\gamma}}{1-\sqrt{\gamma}} \quad (3)$$

$$\beta^{(2)}(\Box) = \frac{1}{\Box}\left\{ \frac{\gamma-1}{4\sqrt{\gamma}}ln\frac{1+\sqrt{\gamma}}{1-\sqrt{\gamma}} + \frac{1}{2} \right\} \quad (4)$$

$$\beta^{(3)}(\Box) = \frac{1}{48\Box}\left\{ \left(3 - \frac{6}{\gamma} + \frac{3}{\gamma^2} \right)\sqrt{\gamma}\, ln\frac{1+\sqrt{\gamma}}{1-\sqrt{\gamma}} - \frac{6}{\gamma} + 10 \right\} \quad (5)$$

and $\gamma = \frac{1}{1-\frac{4m^2}{\Box}}$.

It was pointed out in [9] that the purely geometric term does not contribute to

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the thermal radiation of the classical static black hole background, provided that the mass of the field is nonzero. Large mass expansion of this term yields

$$R\beta^{(3)}(\Box)R = -\frac{1}{60m^2}R^2 + O(\frac{R^4}{m^4}) \tag{6}$$

Thus if we consider the action

$$S = S_{cl} - k \int \sqrt{-g}R^2, \tag{7}$$

with $k = 1/(240m^2)$, the second term can be thought of as a quantum correction in the above described approximation. The full nonlocal one-loop action, even in this approximation, contains also terms like Q^2 and QR. These terms are responsible for the Hawking radiation of the black hole, as was shown in [9]. In the present work, since we are interested in isolating possible effects of non-thermal character, we ignore these terms.

The equations of motion for the action in (7) are

$$\Box\phi - (\nabla\phi)^2 + \frac{1}{4}R + \lambda^2 - \frac{1}{4}(\nabla T)^2 - \frac{m_0^2}{4}T^2 = 0 \tag{8}$$

$$\Box T - 2(\nabla\phi)(\nabla T) - m_0^2 T = 0 \tag{9}$$

$$e^{-2\phi}\left\{-\frac{1}{4}g^{\mu\nu}R - \phi^{;\mu\nu} + \frac{1}{2}T^{,\mu}T^{,\nu}\right\} -$$
$$k\left\{-g^{\mu\nu}(\frac{1}{4}R^2 + \Box R) + R^{;\mu\nu}\right\} = 0 \tag{10}$$

We will seek static black hole solutions, with trivial scalar field configuration $(T = 0)$. The resulting equations are insensitive in the mass of the scalar field, since a change in k can be compensated by a constant shift of the dilaton. Thus in the analysis that follows we take $k = 1$. Working in the Schwarzschild gauge

$$ds^2 = -gdt^2 + \frac{1}{g}dr^2 \tag{11}$$

the field equations take the form

$$g\phi'' - \frac{1}{4}g'' + g'\phi' - g(\phi')^2 + 1 = 0, \tag{12}$$

$$g'''' - e^{-2\phi}\phi'' = 0, \tag{13}$$

$$2gg'''' - \frac{1}{2}(g'')^2 + g'g''' + e^{-2\phi}(-\frac{g''}{2} + g'\phi') = 0. \tag{14}$$

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where the derivations are with respect to the dimensionless variable $x = \lambda r$. The equation (12) is the dilaton equation and (13, 14) are derived from the equations for the metric components (10) with the use of (8). We are interested in solutions which are asymptotically flat. In the gauge we are working $R(r) = -\lambda^2 g''$. So the requirement for asymptotically vanishing curvature gives two qualitatively different asymptotic forms for the metric g:

$$
\begin{array}{rcll}
g & \to & 1 & , \quad as \quad x \to \infty, \\
or \quad g & \to & Ax + B \quad (A > 0), & as \quad x \to \infty,
\end{array}
$$

where the condition $A > 0$ is required by causality. In the next section we will examine the two cases and derive the black hole solutions in each one.

3 Black Hole Solutions

3.1 Case with asymptotically constant metric

A full analytical solution, in closed form, of the nonlinear system of the equations of motion is not possible. Attempting to get numerical solutions with the desired properties is not straightforward because one has to fix six initial values for the fields (and their derivatives). Even in such an effort there are not physical criteria for detetmining the relative values of the six initial conditions. The method we adopt is partially analytic and partially numerical. In particular the solution of system is written in an iteration form. Althought it is not possible to get the solutions in closed form the iteration procedure permits a full control them. To be more precise, taking the solution at the first iteration step and consequently using the values of the fields at an appropriate point as initial conditions, we get full numerical solutions of the system of equations which respect the requirement of asymptotic linear dilaton vacuum. Fufthermore this method has the advantage that permits exact determination of the ADM mass of the corresponding black holes. In the rest of the subsection we will scetch the method adopted and present the basic features of the solutions that we get.

With the required asymptotic behaviour of the metric, we can see from the dilaton equation that the corresponding asymptotic form of the dilaton field is just linear as expected, that is $\phi_{as} = -x$. Taking into account the asymptotic behaviour of the fields g and ϕ we make the following ansatz

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$$g = e^{y(x)},$$
$$\phi = -x + h(x). \tag{15}$$

Therefore we seek solutions with
$$y(x), \quad h(x) \quad \to \quad 0,$$
$$as \quad x \quad \to \quad \infty.$$

Note that the black hole solutions arising from the classical action (1), with trivial scalar field configuration, are given by

$$y^c(x) = ln(1 - Me^{-2x}),$$
$$h^c(x) = 0. \tag{16}$$

Considering the equations (12) and (14) the corresponding first order system takes the form

$$\frac{d\vec{\mathbf{X}}}{dx} = \mathbf{A}(x)\vec{\mathbf{X}} + \vec{\mathbf{F}}(x, \vec{\mathbf{X}}) \tag{17}$$

where
$$\vec{\mathbf{X}} = (h_0 = h, h_1 = h', \widehat{y}_3 = -e^{-2x}y''', y_2 = y'', y_1 = y', y_0 = y),$$
$$\vec{\mathbf{F}}(x, \vec{\mathbf{X}}) = (0, F, G, 0, 0, 0). \tag{18}$$

The matrix
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 1/4 & 1 & 1 \\ 0 & 0 & -2 & -1/4 & -1/2 & 0 \\ 0 & 0 & -e^{2x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{19}$$

determines the linear part of the system (17). The nonlinearity is hidden in the functions F and G, in (18), with

$$F = (1 - y_0 - e^{-y_0}) + h_1^2 + h_1 y_1 + \frac{y_1^2}{4},$$
$$G = e^{-2x}\left(\frac{5}{4}y_1^4 + \frac{11}{4}y_2^2 + 7y_1^2 y_2\right) + \frac{1}{4}\left(1 - e^{-2h_0 - y_0}\right)(y_2 + 2y_1)$$
$$+ \frac{1}{2}e^{-2h_0 - y_0}y_1 h_1 - \frac{9}{2}y_1 \widehat{y}_3.$$

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The eigenvalues of the matrix $\mathbf{A}$ are $(0, 0, -2, -2, -e^x/2, e^x/2)$. The three negative eigenvalues of this matrix yield the solutions of the linear system which vanish asymptotically. Furthermore one expects that the solutions of the full system, having the desired asymptotic behaviour, depend on three parameters too. The system (17) can be written further in integral form

$$\vec{\mathbf{X}}(x) = \mathbf{Y}(x)\vec{\mathbf{C}} + \mathbf{Y}(x) \int_{x_0}^x \mathbf{Y}^{(-1)}(t)\vec{\mathbf{F}}(t, \vec{\mathbf{X}}(t))dt \tag{20}$$

where $\mathbf{Y}$ is a fundamental solution of the linear system satisfying the equation

$$\frac{d\mathbf{Y}(x)}{dx} = \mathbf{A}(x)\mathbf{Y}(x)$$

and $\vec{\mathbf{C}}$ are the corresponding constants of integration. The above integral form dictates the iterative procedure. At each step of the iteration the solution is

$$\overrightarrow{\mathbf{X}^{(n)}}(x) = \overrightarrow{\mathbf{X}^{(0)}}(x) + \mathbf{Y}(x) \int_{x_0}^x \mathbf{Y}^{(-1)}(t)\vec{\mathbf{F}}(t, \overrightarrow{\mathbf{X}^{(n-1)}}(t))dt \tag{21}$$

where $\overrightarrow{\mathbf{X}^{(0)}}$ stands for the general solution of the linear system which has the required asymptotic behaviour.

The linear part of the system (17), as can be easily seen, is equivalent to a system of two algebraic relations

$$h_1(x) = be^{-2x} + \frac{1}{2}y_0(x) + \frac{1}{4}y_1(x), \tag{22}$$

$$\hat{y}_3(x) = -\frac{a}{2}e^{-2x} - \frac{1}{4}y_1(x) \tag{23}$$

and a second order differential equation

$$y_1'' - \frac{e^{2x}}{4}y_1 = \frac{a}{2}. \tag{24}$$

Notice that the constants a, and b are related to the double eigenvalue (-2) of the matrix $\mathbf{A}$. The general solution of the equation (24) is

$$y_1(x) = d_1 K_0(\frac{e^x}{2}) + d_2 I_0(\frac{e^x}{2}) + a\left[A(x) - \frac{x}{2}K_0(\frac{e^x}{2})\right], \tag{25}$$

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where K_0, I_0, are the modified Bessel functions of order zero and $A(x)$ is defined as

$$A(x) = \frac{1}{2}\left\{-I_0(\frac{e^x}{2})\int_x^\infty dt K_0(\frac{e^t}{2}) - K_0(\frac{e^x}{2})\int_{-\infty}^x dt\left[I_0(\frac{e^x}{2}) - 1\right]\right\}. \quad (26)$$

From the asymptotic behaviour of the Bessel functions and their integrals one can see that the parameters d_1 and d_2 correspond to the eigenvalues $-e^x/2$ and $e^x/2$ of the matrix $\mathbf{A}$, respectively, and that

$$A(x) \sim -2e^{-2x} + O(e^{-4x}) \quad (27)$$

All other fields can be derived from (25) by differentiation or integration, or through the algebraic relations (22, 23). Thus we conclude that the solution $\overrightarrow{\mathbf{X}^{(0)}}$ depends on three parameters a, b and d_1. Especially for the components y_0, h_0 and h_1 we get the asymptotic behaviour

$$y_0^{(0)}(x) = ae^{-2x} + O(e^{-4x}) \quad (28)$$

$$h_0^{(0)}(x) = \frac{-b}{2}e^{-2x} + O(e^{-4x}) \quad (29)$$

$$h_1^{(0)}(x) = be^{-2x} + O(e^{-4x}). \quad (30)$$

Taking also into account the asymptotic behaviour of the integral kernel $\mathbf{Y}(x)\mathbf{Y}^{-1}(t)$ of equation (21), after some algebraic work, one can verify that all the fields get contributions of order e^{-4x} at the first iteration step. Note that the above iterative method applied in the case of the classical equations of motion yields the solution in closed form

$$y_0(x) = \sum_{n=0}^{\infty} y_0^{(n)} = ln(1 + ae^{-2x}), \quad (31)$$

where $-a$ is the mass of the black hole.

In order to derive full solutions of the system we take the solution at the first step of the iteration procedure. The corresponding expressions are not given here since they are quite complicated. Then we use the values of the fields obtained in this way, at a point that they approach the corresponding values of the flat space, as initial conditions and subsequently we solve numerically the system. We indeed find solutions with the required asymptotic behaviour. This is due to the stability of the iteration ptocedure. The solution satisfy

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also the equation (13) that was not used so far verifying the compatibility of the full system of the equations of motion. Moreover these solutions for quite natural values of the parameters (this notion will be explained later) exhibit black hole characteristics. In particular they have an horizon at the point that the metric function g vanishes. In figures 1 and 2 we give the plots of the metric and the dilaton fields respectively for a typical structure coming out from the numerical solution of the system of the equations of motion.

Let us proceed by discussing the physical characteristics of the black holes derived. All these black holes have the same temperature with the classical solutions of the CGHS model [2]

$$T = \frac{\lambda}{2\pi}. \tag{32}$$

This holds since the corresponding backgrounds share the same asymptotic behaviour, and can be easily seen from the periodicity of the compactified time coordinate in Euclidean signature. Another way to find the value of the temperature is using the relation

$$T = \frac{\lambda}{4\pi} \left| \frac{dg}{dx} \right|_{x=x_H}, \tag{33}$$

where x_H denotes the position of the horizon. The assignment

$$z = \int^{2x} \frac{1}{g(2x)}$$

transforms the Schwarchzild gauge to the conformal one. In the conformal gauge the apparent horizon is determined from the value $1/2$ of the derivative of the conformal factor. For the static geometries that we discuss here the apparent horizon coincides with the event horizon. Thus the derivative in (33) calculated via its relation with the conformal gauge yields the value in (32).

The ADM mass of a static black hole, approaching asymptotically the linear dilaton vacuum, is given by [1]

$$M = lim_{x \to \infty} \left\{ 2e^{2x} \left[h_1(x) + \frac{1}{2}(e^{-y_0(x)} - 1) \right] \right\}. \tag{34}$$

This formula is exactly applicable in the case that we examine here. For the classical solution, with the values of the fields y_0 and h_1 given in (16) the

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above formula leads to the mass M (in units of the cosmological constant). In the quantum case we see that the linear part of the solution is sufficient to determine the black hole mass. Inserting in (34) the forms of h_1 and y_0 given in (28, 30) we find that

$$M = 2b - a. \tag{35}$$

The above result is exact since it is already pointed out (see also 27) that the first iteration step and therefore all the steps in the iteration procedure give corrections to the fields of order at least e^{-4x}. These corrections do not contribute to the ADM mass. Thus we see that all solutions derived in this way are of finite ADM energy. The natural values of the parameters a and b refered above are those that keep the ADM mass non-negative.

The solutions of the classical equations of motion, with trivial scalar field background, given in (31) depend only on the parameter a. Negative values of a give black hole solutions with mass $-a$ while $a = 0$ is just the flat space (in all these cases the dilaton is linear). In the presence of the R^2 term we have derived solutions that depend on two more parameters (b and d_1). From the triparametric family of solutions the class with $b = d_1 = 0$ are corrections of the classical geometry. These corrections do not alter the physical characteristics of the black hole, as is seen from (32,35). Furthemore for given a the presence of the quantum R^2 term, yields black hole solutions with mass different from the mass of the corresponding classical black hole ($b \neq 0$). We postpone the more detailed discussion about the interpretation of these solutions as backreaction effects for the next section.

Before closing this subsection let us note that the black hole solutions derived are stable against scalar field perturbations. This can be seen by applying the method invoked in [12, 13], for the study of the stability of the black hole solutions in the two-dimensional dilaton gravity. In particular the equation satisfied by the space part of the scalar field, after separation of variables, is Schrödinger-like. Negative eigenvalues of the equation correspond to growing modes of the scalar field perturbations indicating unstability of the background geometry. In figure 3 the potential (U) of the eigenvalue equation is shown. Clearly there are not negative eigenvalues in this potential, justifying the assertion of stability. The form of the potential shows the repulsive character of the R^2 terms. Note that analogous solutions that have been found in [14] four dimensions turned out to be unstable [15].

3.2 Case with asymptotically linear metric

As in the previous subsection using the asymptotic form of the metric we determine from the dilaton equation (12) the asymptotic form of the dilaton field, which reads

$$e^{-2\phi_{as}} = c_1 I_0(\frac{1}{A}\sqrt{B + Ax}) + c_2 K_0(\frac{1}{A}\sqrt{B + Ax}) \qquad (36)$$

The procedure adopted in the former case cannot be followed easily here since an ansatz analogous to (15) is not obvious in this case. This makes the iteration procedure quite complicated. Instead we proceed as follows. Using the asymptotic form of the dilaton we integrate succesively the equation (13). In this way we get more accurate forms of the metric and its derivatives in the asymptotic region. The corresponding values at some point are taken as the initial values for a numerical solution of the system. We find that there exist solutions with the abovementioned asymptotic behaviour, which exhibit the characteristics of a black hole. Namely as in the previous case the function g vanishes at some point indicating thus the position of the horizon. Such kind of solutions, which are asymptoticaly Rindlerian black holes, have already been found in the context of other, different models of dilaton gravity in two dimensions [16, 17]. In general these black holes appear as the end point of gravitational collapse of localized matter. In our case the solutions derived cannot be considered as backreaction effects since their asymptotic behaviour is completely different from that of the classical black holes. In particular besides the linear behaviour of the metric the solutions are strongly coupled ($\phi \to \infty$ as $x \to \infty$). As is seen from (13, 36) only the choice $c_1 = 0$, lead to black hole solutions. In figure 4 we show the behaviour of the function g and the dilaton field in this case. These black hole solutions turn out to be stable against scalar field perturbations, as in the previous case. In general the stability of all the black hole solutions we have found is among the characteristics of the two-dimensional consideration. As is pointed out in [15] this feature does not persists in four dimensions.

4 Discussion - Conclusions

In this work we found the static black hole solutions, with asymptotically vanishing curvature, in the context of two-dimenasional gravity, modified by

the presence of an R^2 term. This term is shown to arise from the one-loop effective action of a massive scalar field, coupled to the theory, in its large mass expansion. The characteristic of this term is that it does not contribute to the Hawking radiation of the classical black hole background.

Two kinds of black hole solutions exist in this model differing in their asymptotic behaviour. The first describes symptotically Rindlerian black holes. These solutions turn out to be strongly coupled, in the sense that the dilaton field diverges in the asymptotic region. There is no connection with the classical black hole backgrounds of the two-dimensional dilaton gravity. So they are not considered to describe quantum correction or the end point of the evolution of a classical black hole.

The second class approaches asymptotically the linear dilaton vacuum. The solutions of this kind depend on three parameters (a, b and d_1). The parameter a characterizes the classical solutions, determining the ADM mass of the corresponding black hole geometries. From the new solutions, in the presence of the R^2 term, the subclass with $a < 0$, $b = d_1 = 0$ have the same physical characteristics of the associated classical black hole. These solutions are backreacted ones. This backreaction is compatible with the fact that the term added in the action does not contribute to the Hawking radiation effect. That is for any classical black hole background there is a new one of the quantum corrected action with the same mass. The correction appears as a shift of the horizon to larger values of x. In these solutions, as can be seen from the continuation of the metric behind the horizon of by transforming to the conformal gauge, the physical singularity appears at finite value of x. Moreover as is seen in figure 2 the dilaton ceases from being monotonic in the vicinity of the horizon. This behaviour accounts for the repulsive nature of the R^2 terms [15], a feature that is also shown in the plot of the potential (fig. 3) arising in the stability analysis.

Besides this class of solutions the presence of the R^2 term may turn on the parameters b and d_1 also. From these parameters, for given a, the parameter b represents the contribution of the dilaton field to the ADM mass of the black hole (such a contribution is absent in the classical case). The numerical analysis shows that there are black hole solutions with either positive or negative b. Of course a complete scenario of the black hole formation and evolution, that exists in the context of the RST model [3], is missing in our case. Nevertheless it is quite natural to assume that there exist time dependent solutions interpolating between a classical or a backreacted solution

$(a < 0,\ b = d_1 = 0)$, and solutions with $b \neq 0$ (more accurately $b = b(t)$, $d_1 = d_1(t)$). Thus in this model evolution of a black hole in the presence of a quantum term which does not give the usual thermal effect can in principle be described.

From the evolution proccesses which may be described, under the above mild assumption, those with $b > 0$ represent the absorption of energy by the black hole. The initial and final states have the same temperature (32) but the mass of the final black hole is increased, according to (35). Among the solutions with $b > 0$ those with $a = 0$ describe the formation of a black hole in the presence of the R^2 term. The inverse proccess is more interesting, where the black hole may lose part of its mass $(b < 0)$. This is achieved through non-thermal signals. Thus it is possible to get some information out of a black hole if the decreasing of its mass is due to the presence of terms of the kind considered so far. The question that arises here is whether this evolution may be responsible for a complete disappearance of a black hole, or differently how far can the mass of the black hole descent down in the presence of the term R^2. The answer to this question seems to be that this evolution is 'soft'. The black hole may lose part of its energy but we cannot argue, from our analysis, that it can reach to a naked singularity. At first our assumption of the time dependent solution interpolating between two static ones is most likely to hold if the difference in the masses of the initial and final configurations is not large $(|b| \ll |a|)$. On the other hand even if we relax this condition we see that chosing $b = a/2$, we still have a black hole structure with vanishing ADM mass. This means that the model we consider accepts massless black holes. The interpretation of structures with negative ADM energy (if we let b to become more negative) is obscure to us. Of course in order to understand further the energy balance and therefore the permitted range of the parameter b, (given the value of a) one has to consider solutions with nontrivial scalar field configuration.

The entropy of the one-loop solutions that we have derived can be calculated by the method described in [18, 19], althought the formula given in [19] is a simple choice for the entropy, due to ambiguites of the Wald construction in the presence of the R^2 term. The result is that it is enhanced , on the event horizon, relative to the entorpy of the classical black hole solutions. This increase is mainly due to the behaviour of the dilaton field near the horizon as is seen in figure 2. The increasing of the entropy is in agreement with the interpretation of the solutions as incoprorating backreaction effects.

In general it is difficult to follow the evolution of a black hole in this model. A better understanding could be achieved by studying the solutions with nontrivial scalar field, as is already refered. From our analytical consideration we see that the matrix **A** in the linear part of the system in (17), extended to include the equation of motion of the scalar field, possesses one more negative eigenvalue, associated with T. Thus it is most likely that, as happens in the classical case [20, 13], static solutions with non-trivial scalar field configuration exist. Furthermore the stability under scalar field perturbations indicate that time dependent solutions, which may appear in the evolution, exist also. Besides this the consideration of the terms Q^2 or QR, which appear in the one-loop effective action in the same approximation, will be helpfull. These terms are responsible for the Hawking radiation of the classical black hole background. Discrimination between the two cases could lead in a better understanding of the quantum effects in the background of a black hole.

Acknowledgements

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Figure Captions

Fig. 1 The behaviour of the metric for the black hole solutions that approach asymptotically the linear dilaton vacuum. The position of the horizon is shown at the point where the function g vanishes. The particular solution presented is determined by $a = -10$, $b = 1$, $d_1 = 10$.

Fig. 2 The behaviour of the dilaton field shows the inclination from linearity in the vicinity of the horizon.

Fig. 3 The potential arising in the stability analysis of the black hole solution presented in figure 1. The barrier form shown exludes negative eigenvalues of the corresponding Schrödinger equation. This means that a scalar field wave packet does not develop growing modes.

Fig. 4 The solid line shows the metric and the dashed line the dilaton field for the asymptotically Rindlerian solutions. The metric grows linearly while the dilaton behaves asymptotically as $\sqrt{x}$.

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